Chapter 7: Potential Energy and Energy Conservation

Goals for Chapter 7

- To understand the concept of the potential energy
- To understand the relationship between work done and the potential energy
- To use gravitational potential energy in vertical motion
- To use elastic potential energy for a body attached to a spring
- To solve problems involving conservative and nonconservative forces
- To determine the properties of a conservative force from the corresponding potential-energy function
- To use energy diagrams for conservative forces
About Potential Energy

- Potential energy
  - The energy stored within a physical system as a result of the position or configuration of the different parts of that system.
  - It has the potential to be converted into other forms of energy, such as kinetic energy, and to do work in the process.
  - The potential energy is a function of the state a system is in, defined relative to an arbitrary reference energy.

1. Gravitational Potential Energy

- Work done by gravity
  \[ W_{\text{gravity}} = \]

- Gravitational potential energy
  (near Earth’s surface)
  \[ U = mgy \]

- Then,
  \[ W_{\text{gravity}} = \]
Conservation of Mechanical Energy

\[ W_{\text{gravity}} = W_{\text{tot}} = \Delta K = -\Delta U \]

\[ K_2 - K_1 = -(U_2 - U_1) \]

\[ \Rightarrow K_1 + U_1 = K_2 + U_2 \]

\[ E = K + U = \text{constant} \]

For gravitational force

\[ E = K + U \]

\[ = \frac{1}{2} mv^2 + mgy \]

Since \( W = -\Delta U \) and \( W = \int F \cdot d\vec{r} \), the potential energy is

\[ U(r) - U(r_{\text{ref}}) = U(r) = -\int_{r_{\text{ref}}}^{r} F \cdot d\vec{r} \]

- \( r_{\text{ref}} \) is the position where the potential energy is zero, i.e., \( U(r_{\text{ref}}) = 0 \)

For gravitational force at the Earth’s surface

\[ U = -\int_0^y (-mg)dy = mgy \]
• Example 1) A baseball is thrown from the roof of 22.0-m tall building with an initial velocity of magnitude 12.0 m/s and directed at an angle of 53.1° above the horizontal.

  a) What is the speed of the ball just before it strikes the ground?

  b) What is the answer for part (a) if the initial velocity is at an angle of 53.1° below the horizontal?
Example 2) A skater moves through a quarter-circle with radius $R=3.00$ m. The mass of the skater and his skateboard is 25.0 kg. Assume that the friction is negligible. Find the normal force that acts on him at the bottom of the curve.
Example 3) A small rock with mass 0.12 kg is fastened to a massless string with length 0.80 m to form a pendulum. The pendulum is swinging so as to make a maximum angle of 45° with the vertical.

a) What is the tension in the string when it makes an angle of 45° with the vertical?

b) What is the tension in the string as it passes through the vertical position?
2. Elastic Potential Energy

- Force by a spring: Hooke’s Law
  \[ F = -kx \quad k : \text{spring constant} \]

- Negative sign means the direction of the force is opposite to the direction of displacement (restoring force).

- From \( U = -\int_{r_{\text{ref}}}^{r} \vec{F} \cdot d\vec{r} \), the potential energy by a spring (restoring force) becomes
  \[ U = -\int_{0}^{x} (-kx)dx = \frac{1}{2}kx^2 \]

- Mechanical energy for mass-spring system
  \[ E = K + U \]
  \[ = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \]
Example 4) A glider with mass 0.200 kg sits on a frictionless horizontal air track, connected to a spring with spring constant $k=5.00 \text{ N/m}$. When the displacement is 0.100 m, it is released. What is the $x$ velocity when $x=0.080 \text{ m}$?
Example 5) A 2.50 kg mass is pushed against a horizontal spring of force constant 25.0 N/cm on a frictionless air table. The spring is attached to the tabletop, and the mass is not attached to the spring in any way. When the spring has been compressed enough to store 11.5 J of potential energy in it, the mass is suddenly released from rest.

a) Find the greatest speed the mass reaches.

b) What is the greatest acceleration of the mass?
Example 6) A 2.00 kg block is pushed against a spring with negligible mass and force constant $k=400$ N/m, compressing it 0.220 m. When the block is released, it moves along a frictionless, horizontal surface and then up a frictionless incline with slope.

a) What is the speed of the block as it slides along the horizontal surface after having left spring?

b) How far does the block travel up the incline before starting to slide back down?
3. Conservative and Nonconservative Force

- **Conservative force**
  - It is reversible.
  - It can always be expressed as difference between the initial and final values of a potential energy function.
  - It is independent of the path of the body and depends only on the starting and ending points.
  - When the starting and ending points are the same, the total work is zero.
  - $E = K + U$ is always constant.
  - The work of the conservative force can always be represented by a potential-energy function.

- **Nonconservative force**: A force that is not conservative
  - The work done by a nonconservative force cannot be represented by a potential-energy function.
  - The work done by a nonconservative force manifests itself as changes in the internal energy of the bodies.
  - Internal energy: The energy associated with the change in the state of the materials.

- **Law of conservation of energy**
  $$K_i + U_i + W_{other} = K_f + U_f$$
Example 7) A block with mass 0.500 kg is forced against a horizontal spring of negligible mass, compressing the spring a distance of 0.200 m. When released, the block moves on a horizontal table top for 1.00 m before coming to rest. The spring constant $k$ is 100 N/m. What is the coefficient of kinetic friction, $\mu_k$, between the block and the table?
Example 8) In a truck-loading station at a post office, a small 0.200 kg package is released from rest at point A on a circular track. It slides down the track and reaches point B with a speed of 4.80 m/s. From point B, it slides on a level surface a distance of 3.00 m to point C, where it comes to rest.

a) What is the coefficient of kinetic friction on the horizontal surface?

b) How much work is done on the package by friction from A to B?
4. Force and Potential Energy

- Potential energy from a conservative force

\[ U = -\int_{r_{ref}}^{r} \vec{F} \cdot d\vec{r} \]

- Force from a potential energy

\[ \vec{F} = -\nabla U = -\left( \frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} + \frac{\partial U}{\partial z} \hat{k} \right) \]